Sample paper for graduate admission test for EE7 - Control and Optimization stream

Department of Electrical Engineering
Indian Institute of Technology Madras, Chennai-36

1. In the following matrix, the equality $a + b = c + d$ holds.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

   (a) Is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector?

   (b) Find both the eigenvalues.

2. Verify if the following functions are differentiable.
   - $|x|^2$
   - $|x|^3$

3. Consider two matrices $A, B \in \mathbb{R}^{n\times n}$. Analyze if the following statement are true or false. If true give reasons/proof and if false give a counter example.
   - The eigenvalues of $A + B$ is same as the sum of eigenvalues of A and B.
   - The eigenvalues of AB is same as the product of eigenvalues of A and B

   In case any of your above answer was false, can you think of some special A and B for which the above condition is true.

4. The open-loop transfer function of a unity-feedback system is given by $G(s) = \frac{K}{s(\tau s+1)}, K, \tau > 0$. By what factor should the gain K be reduced so that the peak overshoot for a unit-step response of the system is reduced from 75% to 25%?

5. Let

$$f(x) = \begin{cases} e^{-\frac{1}{x}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

   - Is $f(x)$ continuous at $x = 0$?.
   - Is $f(x)$ differentiable at $x = 0$?, if so, what is the derivative at $x = 0$.

6. Check if all the roots of $s^3 + 7s^2 + 25s + 39 = 0$ have real parts more negative than -1.

7. The open-loop transfer function of a system is given by

$$G(s) = \frac{K}{s(s+2)(s+4)}.$$

   What is the value of K, which will lead to sustained oscillations in the closed-loop unity feedback system? Find the corresponding frequency of oscillation at that gain.

8. The impulse response of a system is $5e^{-10t}u(t)$, where $u(t)$ is the unit step function. Find its step-response?

9. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, find $A^5 + A^3$.

10. A unity feedback system has open loop transfer function $G(s) = \frac{1}{s(3s+1)(s+2)}$. Find the phase crossover and gain crossover frequencies.

11. Determine the transfer function from the frequency response plot shown in figure 1. Assume the system is minimum phase.

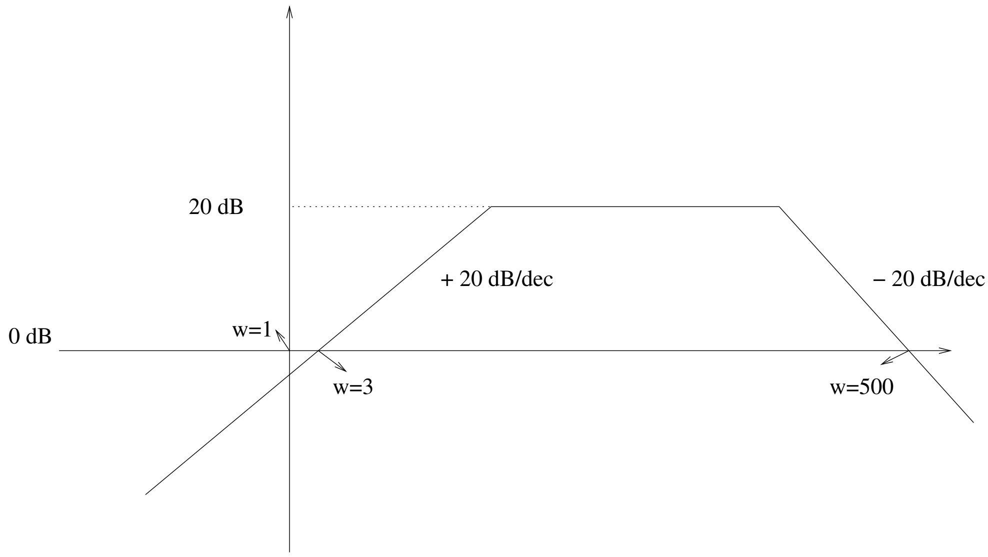


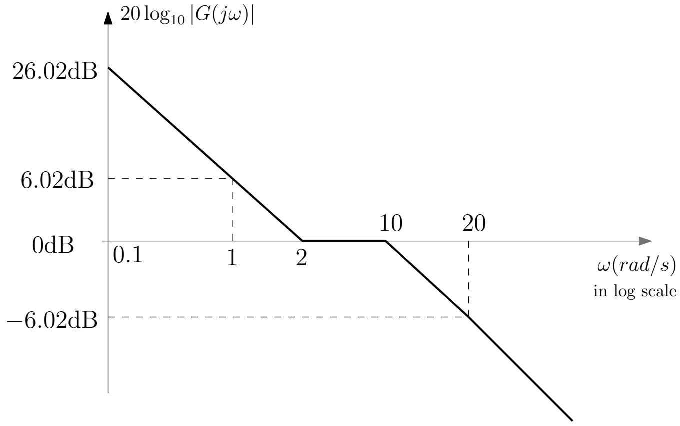
Figure 1: Magnitude plot of $G(j\omega)H(j\omega)$

Sample MS-PhD Entrance Exam

EE7 - Control and Optimization

Date of Examination: June 2020 Max. marks: 30

1. **[5 Marks, 3 + 2]** The Bode asymptotic magnitude plot of a minimum phase system is shown below. The system is connected in a unity negative feedback configuration.



   (a) The corresponding minimum phase transfer function is
      i. $G(s) = K\frac{(1+\frac{s}{2})}{s(1+\frac{s}{10})}$ [correct]
      ii. $G(s) = K\frac{(1+\frac{s}{10})}{s(1+\frac{s}{2})}$
      iii. $G(s) = K\frac{(1+\frac{s}{2})^2}{s^2(1+\frac{s}{10})^2}$
      iv. None of the above

   (b) Calculate the steady state error of the closed loop system, to a unit ramp input. ________ [Answer: 0.5; Range: 0.4 - 0.6]

2. Consider the following nonlinear system in state space form

$$\dot{x}_1 = \mu x_1 - x_1^2$$
$$\dot{x}_2 = -x_2, \quad \mu \in \mathbb{R}.$$

   (a) The number of equilibrium points of the system when $\mu = 0$ are ________ [Ans = 1; Range = 0.98 to 1.02]

   (b) The number of equilibrium points of the system when $\mu > 0$ are ________ [Ans = 2; Range = 1.98 to 2.02]

   (c) The number of equilibrium points of the system when $\mu < 0$ are ________ [Ans = 2; Range = 1.98 to 2.02]

   (d) Which of the following statements is true for $\mu \neq 0$
      i. The system has a unique stable equilibrium point.
      ii. The system has a unique unstable equilibrium point.
      iii. The system has one stable and one unstable equilibrium point. [Correct Answer]
      iv. The system has no equilibrium points.

3. Find the matrix **A** corresponding to the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^3$ such that

$$f(z) = 2x + 3y + 5z$$
$$f(y + z) = x$$
$$f(x + y + z) = y - z$$

where $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3$? [Hint: Use properties of linearity on f]

(a) $\mathbf{A} = \begin{bmatrix} -1 & 1 & -1 \\ -1 & -3 & -5 \\ 2 & 3 & 5 \end{bmatrix}$ [Correct]

(b) $\mathbf{A} = \begin{bmatrix} -1 & -1 & 2 \\ 1 & -3 & 3 \\ -1 & -5 & 5 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 2 & 3 & 5 \end{bmatrix}$

(d) $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

4. What is the transformed vector of $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as per the transformation in Q3 ?

   (a) $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

   (b) $\begin{bmatrix} -1 \\ -9 \\ 10 \end{bmatrix}$ [Correct]

   (c) $\begin{bmatrix} 1 \\ 9 \\ -10 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 \\ 0 \\ 10 \end{bmatrix}$

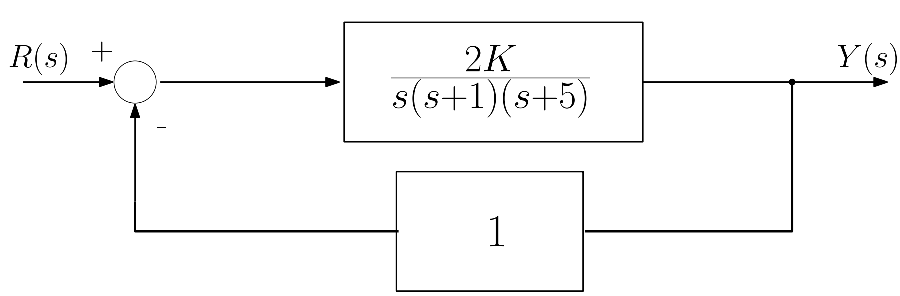
5. The trace and determinant of a 3×3 non-singular matrix $\mathbf{A}$ are 8 and 10 respectively. Which of the following could be the eigen values of $\mathbf{A}^{-1}$?

   (a) 1, 0.5, 0.2 [Correct]
   (b) 1, 4, 4
   (c) 0.5, 0.25, 0.25
   (d) 1, 2, 5

6. The steady-state error is zero for a step input for the system with type

   (a) 0
   (b) 1 [Correct]
   (c) 2 [Correct]
   (d) All of them

7. With a suitable choice of K, the system will oscillate at frequency ω.



The value of K is ____________ and the value of ω is ____________. [Answer: 15 , Range: 14.5 to 15.5, Answer: 2.23, Range: 2.1 to 2.35]

8. Consider a unity feedback system with

$$G(s) = \frac{K(s-a)}{s(s-b)}.$$

Suppose $a < b < 0$**.** What is the complete range of K for which the closed-loop system is stable?

(a) $K > 0$ [correct]
(b) $K > |a|$
(c) $K > |b|$
(d) No value of K.

9. A unity feedback system with

$$G(s) = \frac{K}{s(s+\sqrt{2K})}.$$

For what range of $K > 0$ is the settling time less than 1 second for 3% tolerance?

(a) $K > 18$
(b) $K < 18$
(c) $K > 32$ [correct]
(d) $K < 32$.

10. Consider three linearly independent vectors u, v, w in $\mathbb{R}^3$. Which of the following is true?

(a) u is a linear combination of v and w
(b) u, v and $u+v$ are linearly independent
(c) $au + bv + cw = 0$ for some non-zero constants a, b, c
(d) The vectors u, $u+v$ and $u+v+2w$ are linearly independent [correct]

11. Consider the closed-loop transfer function of a certain plant $H(s) = \frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$. The angular frequency at which the magnitude response is maximum (resonant frequency ω_r) and the response at the resonant frequency (resonant peak $|H(j\omega_r)|$) are

(a) $\omega_r = \omega_n^2$ and $|H(j\omega_r)| = \frac{\omega_n^2}{\zeta}$
(b) $\omega_r = \omega_n$ and $|H(j\omega_r)| = \frac{\omega_n}{2\zeta}$ [correct]
(c) $\omega_r = \frac{\omega_n}{2}$ and $|H(j\omega_r)| = \frac{\omega_n^2}{2\zeta}$
(d) $\omega_r = 2\omega_n$ and $|H(j\omega_r)| = \frac{\omega_n}{2\zeta}$

12. The number of select lines required for a 1-to-8 demultiplexer is

(a) 2
(b) 3 [correct]
(c) 4
(d) 5

13. Let A and B be the inputs to a half-subtractor. The difference output will be

(a) A OR B
(b) A XOR B [correct]
(c) A AND B
(d) A XNOR B

14. For the following polynomial use Routh Hurwitz criteria to identify the number of poles in right half complex-plane (RHP), left half complex-plane (LHP) and imaginary axis ($j\omega$)

$$P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$$

(a) RHP-2 ,LHP-3,$j\omega$-0.

(b) RHP- 1,LHP-2,$j\omega$-2.

(c) RHP-1 ,LHP-4,$j\omega$-0.

(d) RHP-2 ,LHP-1,$j\omega$-2. [correct]

15. Answer if the following statements are true or false.

    (a) A system with positive gain and phase margin is always stable.

    - True
    - False [Correct]

    (b) The state-space representation for a given system is unique.

    - True
    - False [Correct]

    (c) Transfer function of a system depends on the initial conditions in the system.

    - True
    - False [Correct]

16. Figure below illustrates the frequency response of four unknown systems. Which of these systems are minimum phase?

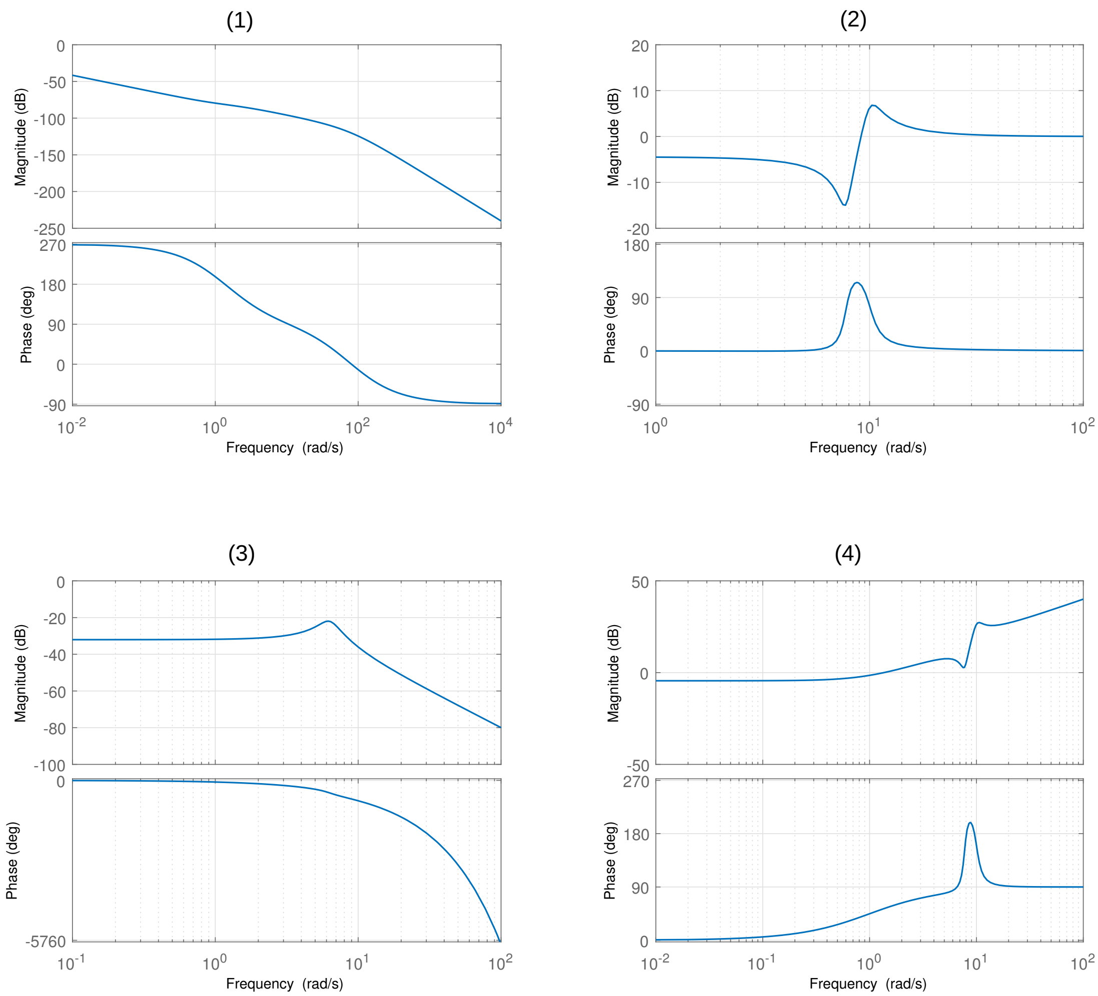


Figure 1: Frequency response of unknown plants

(a) (1) and (2) are minimum phase

(b) (2) and (3) are minimum phase

(c) (2) and (4) are minimum phase [correct]

(d) (3) and (4) are minimum phase

17. A Nyquist plot of a unity feedback system with the feed forward transfer function $G(s)$ is shown in figure below. If $G(s)$ has no pole in the right half s plane, find the range of values for K for which the closed-loop system is stable.

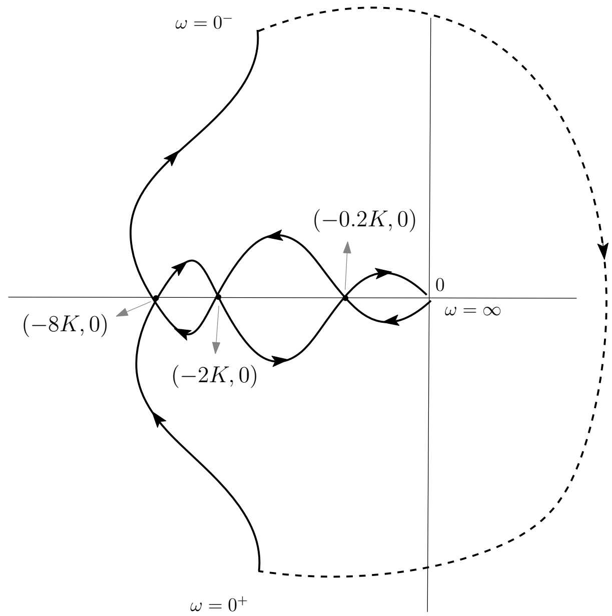


Figure 2: Nyquist plot

(a) $K < \frac{1}{8}$ or $\frac{1}{2} < K < 5$ [correct]
(b) $K < \frac{1}{8}$ and $\frac{1}{2} < K < 5$
(c) $K > \frac{1}{8}$ or $\frac{1}{2} < K < 5$
(d) $K < \frac{1}{2}$ or $K > 5$

18. A Nyquist plot of an open loop transfer function $G(s)$ is given below. Identify the transfer function $G(s)$.

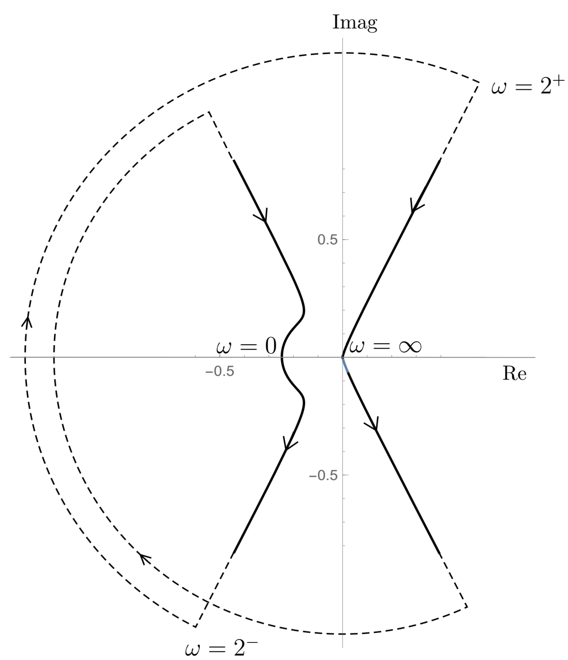


Figure 3: Nyquist plot

(a) $G(s) = \frac{1}{(s^2-4)(s+1)}$

(b) $G(s) = \frac{1}{(s^2+4)(s-1)}$ [correct]

(c) $G(s) = \frac{s}{(s^2+4)(s+1)}$

(d) $G(s) = \frac{s^2}{(s^2-4)}$

MS-PhD Admission Test

EE7 - Control & Optimization

Date of Examination: Nov 2023 Max. marks: 30

Section A: Control Systems

1. **2 marks** The matrix A that represents the linearized model of the following nonlinear system

$$\dot{x}_1 = x_1^2(x_2 - 1) + x_2$$
$$\dot{x}_2 = -x_1 x_2 + x_1$$

on $\mathbb{R}^2$ at the origin $x = 0$ is

a. $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$.

b. $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

c. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

d. $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$.

Solution: *c.*

2. **2 marks** Consider the state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

We design a state feedback controller $u = -Kx$, $K = [k_1 \; k_2]$, such that the closed-loop system has a damping coefficient $\xi = 0.707$ and the step response of the closed loop system has a peak time $t_p = 3.14$ seconds. What is the value of k_1, k_2 in the designed state feedback controller (rounded to the nearest integer)?

Solution:$k_1 = -4, k_2 = -3$

3. **2 marks** A ball is thrown along the x axis The initial position and the initial velocity imparted to the ball are $x(0) = 0$ and $\dot{x}(0) = 150$ Km/hr respectively. The ball follows the dynamics $\ddot{x} = -\dot{x}$. What is the position of the ball in Kilometers at $t = \infty$.
Solution : 150 Km

4. **2 marks** Match the following transfer-functions

(a)

$$G(s) = \frac{3}{s+3}$$

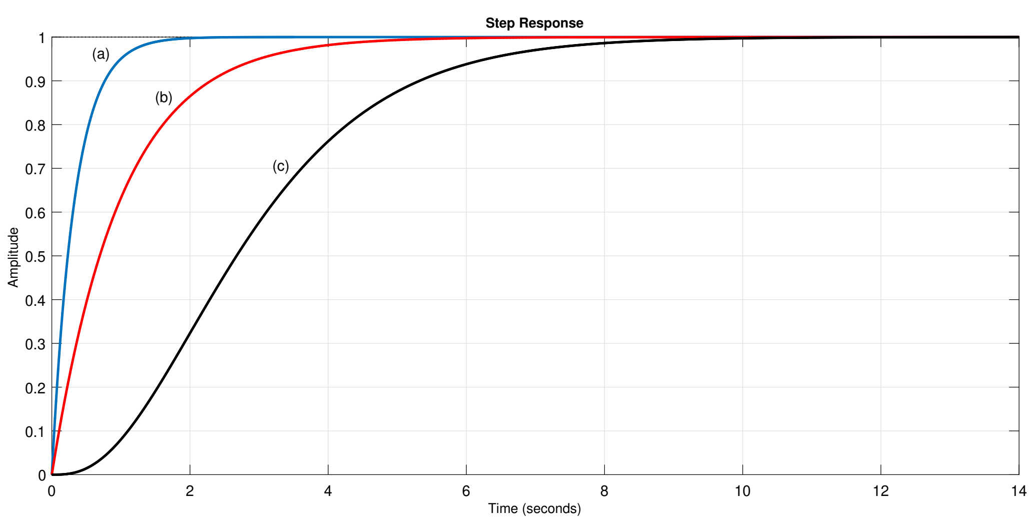
(b)

$$G(s) = \frac{1}{s+1}$$

(c)

$$G(s) = \frac{1}{(s+1)^3}$$

Match the step responses (a), (b) and (c) with their respective transfer-functions given above



Solution: a,b,c

5. **1+1+1 marks** Consider the mapping $f : \mathbb{C} \to \mathbb{C}$, where

$$f(z) \;=\; \frac{1-z}{1+z}.$$

   (a) This mapping transforms the imaginary axis of $\mathbb{C}$ to what ?

   (b) Does the mapping of the imaginary axis encircle the origin ?

   (c) If so how many times ?

   Solution :Unit circle, Yes, -1 times

6. **3 marks** Calculate the 2's complement representation of -37.75 with the smallest number of bits.

   Solution: 1011010.01

7. **3 marks** Consider a 1-bit full-adder. Let the propagation delay between an input and carry be 0.3ns and the propagation delay between input and sum be 0.7ns. Suppose, we construct a 32-bit ripple carry adder using such full adders. What is the worst-case propagation delay between input and output ?

   Solution: 10ns (31 x 0.3 + 0.7)

8. **3 marks** A program has 3 main loops: "Input" loop consisting of 6 instructions, "Compute" with 5 instructions and "Output" with 5 instructions. Each of these loops is run 1 million times and the total execution time is 1 second. One of the instructions in "Compute" is a floating-point multiply taking 5 cycles while the others are non-ALU operations. Assuming a 30MHz clock speed, what is the average MIPS of performance delivered by the system on this program ?

   Solution: 16 (6 + 5 + 5)

9. [**2 marks**] Consider a unity feedback system with

$$G(s) = \frac{K(s-a)}{s(s-b)}.$$

Suppose $a < b < 0$**.** What is the complete range of K for which the closed-loop system is stable?

(a) $K > 0$

(b) $K > |a|$

(c) $K > |b|$

(d) No value of K.

Solution: a

10. [**3 marks**] For the root locus shown in Figure 1, the value of the gain that will make the system marginally stable is

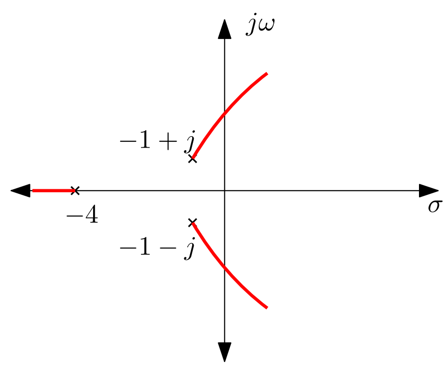


Figure 1: Root locus plot for Question No. 10

Solution:52

Section B: Mathematics

1. Consider the optimization problem

$$\begin{aligned} & \min_{(x,y)} x^2 + y^2 \\ \text{such that} \quad & x - y = 1 \end{aligned}$$

   Find the optimal values (x, y) and the optimal cost function.

   (a) $(1, 0)$ and 1.
   (b) $(\frac{1}{2}, -\frac{1}{2})$ and 1.
   (c) $(0, -1)$ and 1.
   (d) $(\frac{3}{4}, -\frac{1}{4})$ and $\frac{10}{16}$.

   Solution: b

2. The solution set to the equation

$$x + y + z \quad = \quad 0$$

   (a) $S = \{(0, 0, 0)\}$
   (b) $S = \{\alpha(-1, 1, 0) + \beta(-1, 0, 1) \mid \alpha, \beta \in \mathbb{R}\}$
   (c) $S = \{\alpha(1, -2, 1) + \beta(-2, 1, 1) \mid \alpha, \beta \in \mathbb{R}\}$
   (d) None of the above

   solution (b) and (c)

3. **2 marks** For what values of constants a, b is $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} ax + 2b & x \leq 0 \\ x^2 + 3a - b & 0 < x \leq 2 \\ 3x - 5 & x > 2. \end{cases}$$

   continuous at every $x \in \mathbb{R}$?

   a. $a = 0, b = 0$.
   b. $a = 2/3, b = -2/3$.
   c. $a = 1/3, b = 1/3$.
   d. $a = -3/2, b = -3/2$.

   Solution: *d*.

4. **2 marks** A function $f : [0,1] \longrightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} x & x \text{ is} \quad \text{rational} \\ 1 - x & x \text{ is} \quad \text{irrational} \end{cases}$$

Then f is

a. not one-to-one on $[0,1]$ and continuous at every $x \in [0,1]$.

b. one-to-one on $[0,1]$ and continuous only at $x = 1/2$.

c. one-to-one on $[0,1]$ and discontinuous only at $x = 1/2$.

d. Neither one-to-one nor continuous at every $x \in [0,1]$.

Solution: b.

5. **3 marks** Consider a uniformly distributed random variable X with the probability density function $f(x) = 1,\ 0 \leq x \leq 1$. Let Y be another random variable which is defined as $Y = a + bX$, where $a \geq 0$ and $b \neq 0$. Then the probability density function of the random variable Y is given by

a. $g(y) = \frac{1}{|b|} f\left(\frac{y-a}{b}\right),\ y \in \{a + bx,\ x \in [0,1]\}$.

b. $g(y) = \frac{1}{|b|} f\left(\frac{y-a}{b}\right),\ y \in [a,\ a+b] \cup [a-b,\ a]$.

c. $g(y) = \frac{1}{b} f\left(\frac{y-a}{|b|}\right),\ \begin{cases} y \in [a-b,\ a], & b < 0 \\ y \in [a,\ a+b], & b > 0 \end{cases}.$

d. $g(y) = \frac{1}{|b|} f\left(\frac{y-a}{b}\right),\ \begin{cases} y \in [a+b,\ a], & b < 0 \\ y \in [a,\ a+b], & b > 0 \end{cases}.$

Solution: a and d.

6. **3 marks** Two independent random variables X and Y are uniformly distributed in the interval $[-1,\ 1]$. The probability that $X + Y$ is less than 1 is

a. $\frac{6}{8}$.

b. $\frac{7}{16}$.

c. $\frac{7}{8}$.

d. $\frac{9}{16}$.

Solution: c.

7. **2 marks** $A \in \mathbb{R}^{3\times 3}$ has 3 eigenvalues 0, 1, 2. Then

   a. Rank of $A = 0$, Determinant of $A = 2$
   b. Rank of $A = 1$, Determinant of $A = 0$
   c. Rank of $A = 2$, Determinant of $A = 0$
   d. Cannot determine with the given information

   Solution: *c.*

8. **2 marks** Let x, y and z be a linearly independent set of vectors. Which of the following set of vectors are independent?

   (a) $x,\ x+y,\ x+y+z$
   (b) $x+y,\ y+z,\ x+z$
   (c) $x-y,\ y-z,\ z-x$

   a. Only (a)
   b. (a) and (b)
   c. (b) and (c)
   d. None of the above

   Solution: *b.*

9. **2 marks** Let W be the set of all $(x_1, x_2, x_3, x_4, x_5)$ in $\mathbb{R}^5$ which satisfy

$$2x_1 - x_2 + (4/3)x_3 - x_4 = 0$$

$$x_1 + (2/3)x_3 - x_5 = 0$$

$$9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0$$

   Find a finite set of vectors which span W.

   a. $\begin{bmatrix}0 & 2 & \frac{-2}{3} & 1 & 0\end{bmatrix}$, $\begin{bmatrix}0 & 1 & 0 & -1 & 0\end{bmatrix}$ and $\begin{bmatrix}1 & 1 & 0 & 1 & 1\end{bmatrix}$
   b. $\begin{bmatrix}1 & \frac{1}{3} & -2 & -1 & 0\end{bmatrix}$, $\begin{bmatrix}0 & 2 & \frac{-2}{3} & 1 & 0\end{bmatrix}$ and $\begin{bmatrix}0 & 1 & 0 & -1 & 0\end{bmatrix}$
   c. $\begin{bmatrix}0 & 1 & 0 & -1 & 0\end{bmatrix}$ and $\begin{bmatrix}1 & 1 & 0 & 1 & 1\end{bmatrix}$
   d. $\begin{bmatrix}1 & \frac{1}{3} & -2 & -1 & 0\end{bmatrix}$ and $\begin{bmatrix}0 & 2 & \frac{-2}{3} & 1 & 0\end{bmatrix}$

   Solution: *c.*

10. **2 marks** Which of the following subspaces has a finite dimensional basis?

(a) The space $\mathbb{R}$ - Set of all real numbers

(b) The space of polynomials of all degree

(c) The space of polynomials of degree n or less

a. (a) and (b)

b. Only (c)

c. (a) and (c)

d. None of them

Solution: *c*.